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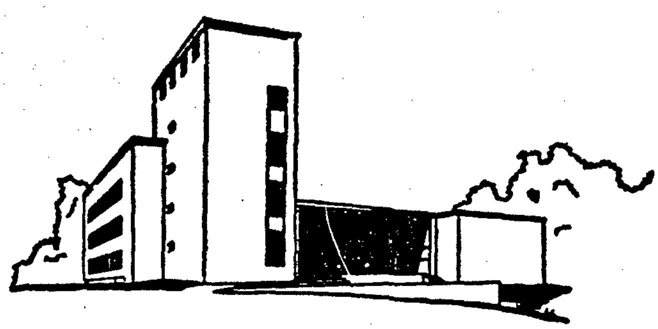

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RESEARCH AND DEVELOPMENT COORDINATION

AS A PROBABILITY PROCESS

by

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Abstract

Models are set up to help decide upon the appropriate scales of Research and Development activities in an organization when competitive preemption of new ideas is a possibility.

I. INTRODUCTION

It is widely, if implicitly, recognized that the production of innovative ideas or products tends to occur in an irregular, apparently random, manner. That is, the establishment of a research activity or laboratory cannot guarantee steady production of profitable inventions hot from the griddle on demand. Rather, such an activity generates a sporadic series of discoveries and new relationships whose commercial implications remain to be established. Furthermore, once the new notion is at hand, a further time of unpredictable duration usually must elapse before it can be "reduced to practice," readied for market, or -- in the case of scientific discoveries--published in a journal.

Suppose that an organization (0 for short hereafter) carries out two activities, Research and Development. The responsibility of Research is the creation or identification of ideas or inventions that are later transformed into marketable products or services by Development. In view of the randomness inherent in the research process, a backlog of inventions will occasionally be present, awaiting the Development stage. If in the mean time competition comes upon a related or better idea and develops it into a marketable product before 0 succeeds in doing so the potential market for the product is likely to be altered, and consequently so is the profit of 0. The chance of such a happening is related to the speed with which the step from invention to marketable product can be executed. In this paper we set up several simple models in an attempt to bring out the relationship between the scales of research and development activities in an organization that must function in such a competitive environment. The problem of timing the introduction of a single new product is treated in [3].

II. FIRST MODEL: AN ORGANIZATION WITH FIXED DEVELOPMENT CAPACITY.

Suppose the research activity of the organization O produces ideas for new products in accordance with a stationary Poisson process of rate λ inventions per year. Each new idea passes to the development activity, where it must await transformation into an actual product. In general a backlog of ideas awaiting development attention will exist; once an idea comes under active consideration the time for its conversion into a product is assumed to be an exponentially distributed random variable with mean μ^{-1}. Furthermore, ideas are dealt with in the order of their appearance.

In order to reflect the presence of competition we assume that if an idea has a total delay (wait plus conversion time) of d then the probability that another company markets the idea first is $1 - e^{-kd}$ $(k > 0)$.

Since our model corresponds to a single server queue with arrival rate λ and service rate μ the long-run total delay density is when $\lambda < \mu$ well-known to be the exponential:

$$f_D(x) = (\mu-\lambda)\, e^{-(\mu-\lambda)x} \qquad x \geq 0$$
$$= 0 \qquad x < 0 \tag{2.1}$$

If $\lambda \geq \mu$ it is obvious that the research activity overwhelms development, and hence many ideas will remain long unexploited or else be preempted by competition. Consequently an attempt must be made to select optimum values of λ and μ such that $\lambda < \mu$.

Note that under more general circumstances, when the imput rate (λ, here) is close to, but still less than, the service rate (μ), an exponential long-run density still prevails, approximately:

$$f_D(x) \simeq e^{-\alpha x}\alpha \qquad x \geq 0$$
$$\simeq 0 \qquad x < 0. \tag{2.2}$$

The parameter α takes the form

$$\alpha = \left\{ \frac{E[A] - E[S]}{Var[S] + Var[A]} \right\} \tag{2.3}$$

where A is the time between two successive arrivals of ideas at the development stage, and S is an idea's conversion time. Approximation (2.2) (the "diffusion" or "heavy-traffic" result; see Kingman [4], and Gaver [2]) turns out to be quite adequate in the event that successive inter-arrivals and services are close to being independently and identically distributed. Consequently, although we shall use the specific model (2.1) to describe delays, it may be anticipated that our specific analysis may be slightly modified to suit broader conditions.

In terms of our model several results follow easily.

(a) The Probability of Competitive Preemption

The chance that a rival company markets a new idea before 0 completes its development is

$$\begin{aligned} P\{\text{preemption}\} &= \int_0^\infty (1-e^{-kx})\, f_D(x)dx \\ &= \int_0^\infty (1-e^{-kx})\, e^{-(\mu-\lambda)x}(\mu-\lambda)dx = \frac{k}{\mu - \lambda + k} \end{aligned} \tag{2.4}$$

(b) Linear Cost Structure

Suppose that research and development expenses per period are linear in the respective production rates:

$$\begin{aligned} \text{Research } (\$/\text{year}) &= C_1\lambda \\ \text{Development } (\$/\text{year}) &= C_2\mu. \end{aligned} \tag{2.5}$$

The expected gain from a project (idea) is denoted by

G_1^1 = gain (\$) if 0 completes development (markets) first

G_2^1 = gain (\$) if competition markets first.

Put

$$G_1 = G_1^1 - C_1 , \quad \text{and} \quad G_2 = G_2^1 - C_1 \quad ; \tag{2.6}$$

normally $G_2 < G_1$. It now follows directly from (2.4) that the expected gain per project equals

$$G_1^1 \frac{\mu - \lambda}{\mu - \lambda + k} + G_2^1 \frac{k}{\mu - \lambda + k} ;$$

the expected number of new projects initiated per period is λ, and the research and development costs are given by (2.5) so the net profit per period is

$$\pi = G_1 \lambda \frac{\mu - \lambda}{\mu - \lambda + k} + G_2 \lambda \frac{k}{\mu - \lambda + k} - C_2 \mu . \tag{2.7}$$

The dependence of net profit rate π on research and development production rates may now be studied by differentiation.

(c) Optimal Research Production (λ) for Given Development (μ).

Suppose development rate, μ, is fixed and a corresponding research rate is to be established. Solution of $\frac{\partial \pi}{\partial \lambda} = 0$ gives

$$\lambda^*(\mu) = (\mu + k) - (1 - G_2/G_1)\, k\, (\mu + k) \tag{2.8}$$

Since the second-order condition $\frac{\partial^2 \pi}{\partial \lambda^2} < 0$ it is clear that we have a relative maximum.

(d) Optimal Development (μ) for Given Research (λ).

This time differentiation of π with respect to μ furnishes the prescription

$$\mu^*(\lambda) = \lambda - k + \lambda k \frac{G_1 - G_2}{C_2} \tag{2.9}$$

and the second-order conditions are fulfilled, showing that a relative maximum is achieved.

Discussion

The simple result of (c) furnishes guidance in the case a firm's development capabilities are essentially fixed, and the proper research level is to be contracted for, perhaps from an external agency. The result (2.8) must turn out to be numerically less than μ, or the model is invalid, and other considerations must be brought to bear. The case of (d), relates to the situation in which a research activity wishes to subcontract for its development activity. Again it is necessary to verify that $\mu^* > \lambda$ in order that the basic model be applicable; a necessary condition is that $\lambda > k\, C_2(G_1-G_2)^{-1}$.

The quantitative behavior of the solution (2.8) and (2.9) conforms to intuition: for example, λ^* increases with k, the rate of competitive activity; if k approaches zero -- a monopolistic situation -- organization O need not spend much on research to avoid competitive preemption.

A more refined model would perhaps call for priority treatment in the development stage for certain especially revolutionary and promising new ideas that should be pushed rapidly into the market to forestall competitive action.

Of some interest is the fact that without furthur constraints, or without the introduction of diminishing returns to scale, there exists no global optimum solution (λ^*,μ^*). This can be shown analytically by investigation of the second-order conditions; in particular $\frac{\partial^2\pi}{\partial\lambda^2} \cdot \frac{\partial^2\pi}{\partial\mu^2} - \left(\frac{\partial^2\pi}{\partial\lambda\partial\mu}\right)^2 < 0$, so a saddlepoint and not a maximum exists. It seems reasonable, however to consider two alternative cost structures that do lead to global optima. These are following:

(i) Convex costs

(ii) Budget constraints.

Consider, for example in case (i) the counterparts to (2.5):

$$\begin{aligned} &\text{Research (\$/year)} = C_1 \lambda^b && (b > 1) \\ &\text{Development (\$/year)} = C_2 \mu^d && (d > 1) \end{aligned} \qquad (2.10)$$

Apparently yearly profit rates increase at most proportionally to λ, while costs rise faster. Consequently global optima appear, and may be found numerically by computer search over the region $\lambda < \mu$.

In the practical event that budget constraints become important then the problem becomes that of selecting λ and μ to maximize π -- see (2.7) -- subject to $C_1 \lambda + C_2 \mu = L$, and $\lambda < \mu$ $(\lambda, \mu > 0)$. Lagrangian techniques apply, the first-order conditions yield quadratics, and so explicit solutions may actually be obtained. Further details are omitted.

III. SECOND MODEL: AN ORGANIZATION WITH BACKLOG-DEPENDENT DEVELOPMENT RATE.

It seems reasonable to alter the previous model in such a manner as to let development activity speed up when backlogs increase. That is, for example, let all innovations be worked on in parallel; then service rate is $\mu_j = j\nu$ if j projects are simultaneously in progress, and $\nu > 0$ is the basic processing rate. Again we assume that innovations occur in a Poissonian fashion, at rate λ, and that competitive action operates at rate k as before.

If j ideas are simultaneously in the development stage, and each is susceptible to competitive preemption, then the rate of this preemption is jk. We assume that once preemption occurs organization 0 ceases further development activity on any related invention or idea $(G_2^1 = 0)$. Needless to say, other formulations are possible -- e.g. 0 may hurriedly engage in hot pursuit to develop a competitive, somewhat different, product. However, we shall limit the present discussion to the simpler version described.

It can be shown that if the total effective processing rate (development plus preemption) is proportional to the backlog, then the long-run distribution of backlog is Poisson:

$$P_j = P\{\text{number of projects undergoing development} = j\} = \frac{\alpha^j e^{-\alpha}}{j!} \quad (3.1)$$

where $\alpha = \lambda(\nu + k)^{-1}$ (see Feller [1], p. 462). The following facts may now be recorded.

(a) The Probability of Competitive Preemption.

Consider the organization at a moment when j development projects are simultaneously in process. Well-known birth and death process properties show that the chance that a project completes in $(t, t + dt)$ is $j(\nu + k)dt$, and hence the probability that the completion is a preemption is $k(\nu + k)^{-1}$.

Hence it follows that

$$\begin{matrix}\text{The long-run rate at which} \\ \text{O completes projects without} \\ \text{preemption}\end{matrix} = \lambda \frac{\nu}{\nu + k} \quad (3.2)$$

(b) Cost Structure.

To represent research costs, let

$$\text{Research (\$/year)} = C_1 \lambda^b \qquad (b > 0); \qquad (3.3)$$

the latter may be specialized to linearity ($b = 1$) or allowed to exhibit decreasing returns to scale ($b > 1$) as required.

Concerning development, we postulate rate change expenses (hiring and firing, for example) that vary with the square of deviations from the average or normal level, m. The total cost of development activity is the sum of a cost proportional to the average effort (or staff level), and a cost associated with the variation in that level. Recalling that when j development activities are in process the production rate is $j\nu$ we are led to consider the two cost rates:

$$\text{Base Development Cost Rate} = C_2\nu\,(\text{Expected number of development projects})$$

$$= C_2\nu \sum_{j=0}^{\infty} j\,p_j = C_2 \frac{\lambda\nu}{\nu + k}$$

$$\text{Development Rate-Change Cost Rate} = C_3\nu^2\,[\mathrm{var}(j)] = C_3\nu^2 \frac{\lambda}{\nu + k} \tag{3.4}$$

It follows that the total profit rate is

$$\pi = G_1^1 \frac{\nu\lambda}{\nu + k} - C_1\lambda^b - C_2 \frac{\nu\lambda}{\nu + k} - C_3 \frac{\nu^2 \lambda}{\nu + k}$$

$$= G_3 \frac{\nu\lambda}{\nu + k} - C_1 \lambda^b - C_3 \frac{\nu^2 \lambda}{\nu + k} . \tag{3.5}$$

where we put $G_3 = G_1^1 - C_2$.

(c) Optimal Rates

In the present model it is possible to solve explicitly for optimal research and development rates, λ^* and ν^* respectively. The necessary condition that $\frac{\partial\pi}{\partial\lambda} = 0$ yields

$$\frac{\partial\pi}{\partial\lambda} = G_3 \frac{\nu}{\nu + k} - C_1\, b\, \lambda^{b-1} - C_3 \frac{\nu^2}{\nu + k} = 0$$

which leads to

$$\lambda^*(\nu) = \left[G_3 \frac{\nu}{C_1 b(\nu+k)} - C_3 \frac{\nu^2}{C_1 b(\nu+k)} \right]^{1/b-1} \tag{3.6}$$

It is straightforward to check that $\frac{\partial^2\pi}{\partial\lambda^2} < 0$, and hence that a (local) maximum is assumed. Now solution of $\frac{\partial\pi}{\partial\nu} = 0$, which turns out to be a quadratic equation in ν, prescribes that

$$\nu^* = k \left[1 + \frac{G_3}{C_3 k} - 1 \right] \tag{3.7}$$

which may be substituted into (3.6). A check of the second-order conditions verifies that a joint maximum is the result.

Discussion

It will be immediately recognized that the explicit solutions (3.6)

and (3.7) flow from the specific cost assumptions and from the control policy that effective development service (processing) rate is $\mu_j = j\nu$. One suggestive line for further investigation would be that of deriving an optimal service function μ_j. Possibly the methods of Markov programming are applicable in this connection. Some simple strategies that appear reasonable, if not globally optimal, might be the following.

(i) The maintenance of rate μ until backlog exceeds $j_o > 0$, at which time a change is made to rate $\mu^1 > \mu$. If backlog ever exceeds $j_1 > j_o$ the excess is sold off or subcontracted for development. The parameters most probably must be selected with with the aid of a numerical search technique.

(ii) At the moment an idea becomes available for development its net market potential, when developed, is assessed; denote this potential by V. Note that the latter may depend upon the actual timing of the introduction as the latter relates to possible competitive introductions. Then if there are N ideas in the development stage, with potentials $V_1, V_2, \ldots, V_N$ priority of emphasis is given to the idea with maximum V-value. Refinements of this type of strategy would recognize the errors of estimate of V.

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